Calculational techniques (not only) for single top production

Stefan Weinzierl[a]

[a]NIKHEF Theory Group, P.O. Box 41 882, NL - 1009 DB Amsterdam, The Netherlands

A next-to-leading order calculation for single top production including spin-dependent observables requires efficient techniques for the calculation of the relevant loop amplitudes. We discuss the adaption of dimensional regularization, the spinor helicity method and of tensor integral reduction algorithms to these needs.

1. Single top production

The top quark, discovered 1995 at Fermilab, is special among the quarks due to its large mass: Its lifetime is shorter than the characteristic hadronization time scale and therefore top bound states do not have time to form. From a calculational point of view this allows an immediate application of perturbation theory to top physics. Up to now top quarks have only been produced in pairs through the strong interaction. With the upcoming Run II at the Tevatron and later with LHC one expects to produce a top quark also through an electroweak Wtb-vertex. The principal production mechanisms for single top production are flavour excitation, W-gluon fusion, s-channel production and associated W-production. Some representative Feynman diagrams are shown in fig. 1. Single top production will be an essential input for a direct measurement of the CKM matrix element V_{tb}. In addition, a discovery of non-standard charged-current top couplings might give a hint on new physics. Since the electroweak decay of the top quark proceeds so rapidly before any hadronization effects can take place, the decay products of the top quark are correlated with the top quark spin. Of particular importance will be the semileptonic decay $t \to b\bar{l}\nu$, since a detector signal corresponding to hadronic top decays will suffer from large QCD backgrounds. Single top production, in which a top quark is produced through a left-handed interaction, offers therefore an opportunity for polarisation studies. Due to the short lifetime of the top quark, production and decay of a top quark should be considered together and the relevant processes at the parton level are for example $u+b \to b+\bar{l}+\nu+d$ for flavour excitation or $u+g \to b+\bar{l}+\nu+d+\bar{b}$ for W-gluon fusion. Here it is understood that we may replace the (u,d)-quark pair by $(\bar{d},\bar{u})$, (c,s) or $(\bar{s},\bar{c})$. Of course most diagrams contributing to the two processes above do not contain a top quark at all. We performed a full leading-order calculation [1] and compared the results to the top narrow width approximation, to which only diagrams with an intermediate top quark contribute. The top narrow width approximation significantly reduces the number of diagrams. Our results for the Tevatron are shown in table 1. The first column gives the results from the full calculation, in the second column we required in addition that the decay products of the top reconstruct to within 20 GeV to

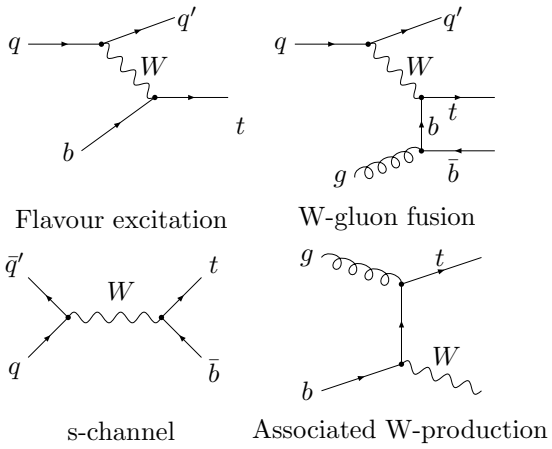


Figure 1. Representative Feynman diagrams for single top production

Tevatron	σ_{tot}	± 20 GeV	narrow width
Wg	15.0 ± 0.4 fb	14.3 ± 0.3 fb	14.5 ± 0.1 fb
Wb	87 ± 1 fb	85 ± 2 fb	87 ± 1 fb
$q\bar{q}$	46 ± 1 fb	32.3 ± 0.3 fb	29.0 ± 0.2 fb

Table 1
Numerical results for Tevatron at 2 TeV for W-gluon fusion (Wg), flavour excitation (Wb) and s-channel production ($q\bar{q}$).

the top quark mass and the third column gives the results for the narrow width approximation. For W-gluon fusion we required three jets, two of them b-tagged, for flavour-excitation two jets with one b-tag and for the s-channel process we required two b-tagged jets. Jets were defined by the hadronic k_T-algorithm [2]. This exclusive signal definition allows us to distinguish the various processes. From table 1 we see that the narrow width approximation describes the cross section very well for W-gluon fusion and flavour excitation. The approximation is less satisfactory for the s-channel process. Here non-resonant terms seem to give a more sizeable contribution. We obtained similar results for the LHC.

2. QCD corrections

A leading-order calculation gives a rough description of the process under consideration, but to reduce ambiguities due to the choice of renormalization or factorisation scales a next-to-leading order (NLO) calculation is required. Furthermore, if jets are defined an NLO-calculation models more accurately the internal structure of a jet. QCD corrections to the s-channel process and flavour excitation have been considered in [3,4]. The calculation of QCD correction to W-gluon fusion is in progress [5]. This calculation should give a more reliable prediction on the p_T-spectrum of the $\bar{b}$-jet. Since the top narrow width approximation works well for W-gluon fusion, it is sufficient to calculate QCD corrections in this narrow width approximation. This simplifies the task considerably. The calculation of the relevant loop amplitudes can in principle be done entirely with conventional methods: One approach could be to calculate

$$2 \text{ Re } A^*_{Born} A_{Loop} \tag{1}$$

and to use the Passarino-Veltman algorithm for the reduction of tensor integrals [6]. One uses further the 't Hooft-Veltman prescription [7] for γ_5, or, more efficiently, the reformulation according to S. Larin [8]. Observables depending on spins can be treated within the spin density matrix formalism. This solves the problem of calculating the one-loop amplitudes in principal, but leads in practice to complicated expressions. A more efficient approach is to calculate helicity amplitudes and to square them numerically. Here the complexity grows linearly with the number of diagrams as opposed to a quadratic increase within the conventional approach. The calculation of spin-dependent quantities is trivial, since helicity amplitudes carry the complete spin information. Efficient algorithm for the reduction of tensor loop integrals rely on Fierz and Schouten identities and "four-dimensional" regularization schemes like dimensional reduction [9] or the FDH-scheme [10] are therefore favoured. (Of course these schemes are all variations of dimensional regularization.) Since weak interactions do not conserve parity, γ_5 makes it appearance and one carefully has to avoid inconsistencies inherent in some of the four-dimensional schemes. We first review the spinor helicity method, before constructing a regularization scheme adapted to γ_5, and comment finally on reduction algorithms for tensor integrals.

3. Spinor helicity method

The spinor helicity method expresses the polarisation vectors for external gluons of momentum

k in terms of two-component Weyl spinors $|p\pm\rangle$ and $\langle p\pm|$ as

$$\varepsilon_\mu^+ = \frac{\langle q-|\gamma_\mu|k-\rangle}{\sqrt{2}\langle qk\rangle}, \varepsilon_\mu^- = \frac{\langle q+|\gamma_\mu|k+\rangle}{\sqrt{2}[kq]}. \quad (2)$$

We have used the customary short-hand notation: $\langle ij\rangle = \langle p_i - |p_j+\rangle$, $[ij] = \langle p_i + |p_j-\rangle$. Here q is an arbitrary light-like "reference momentum". The dependence on the choice of q drops out in gauge-invariant amplitudes. In the narrow width approximation we treat the massive top quark as an external state. For a massive spinor we use [11]

$$\begin{aligned} u(p,\pm) &= \frac{1}{\sqrt{2pq}}(\not{p}+m)|q\mp\rangle, \\ \bar{u}(p,\pm) &= \frac{1}{\sqrt{2pq}}\langle q\mp|(\not{p}+m). \end{aligned} \quad (3)$$

Here, p is the four-vector of the massive fermion with $p^2 = m^2$ and $p_0 > 0$, and q is an arbitrary null vector with $q_0 > 0$. It is easy to check that for these spinors the Dirac equations, orthogonality and completeness relations hold. It will be advantageous to choose for q the momentum of the charged lepton from the top decay. This choice simplifies the factorisation formula in the narrow width approximation.

4. Variants of dimensional regularization

We first give an overview of existing dimensional regularization schemes. Naive dimensional regularization uses

$$\{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu}, \qquad \{\gamma^\mu,\gamma_5\} = 0. \quad (4)$$

In the 't Hooft-Veltman scheme [7] a D-dimensional object is split between a four-dimensional part and a (-2ε)-dimensional part:

$$k_\mu^{(D)} = k_\mu^{(4)} + k_\mu^{(-2\varepsilon)}, \qquad \gamma_\mu^{(D)} = \gamma_\mu^{(4)} + \gamma_\mu^{(-2\varepsilon)}. \quad (5)$$

Here γ_5 is defined as a generic four-dimensional object:

$$\gamma_5 = \frac{i}{4!}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu^{(4)}\gamma_\nu^{(4)}\gamma_\rho^{(4)}\gamma_\sigma^{(4)}. \quad (6)$$

As a consequence, γ_5 anticommutes with the first four Dirac matrices, but commutes with the remaining ones:

$$\left\{\gamma_\mu^{(4)},\gamma_5\right\} = 0, \qquad \left[\gamma_\mu^{(-2\varepsilon)},\gamma_5\right] = 0. \quad (7)$$

Dimensional reduction [9] continues the momenta to $D < 4$ dimensions, but keeps spinors and vector fields in four dimensions.

$$\{\gamma_\mu,\gamma_\nu\} = 2g_{\mu\nu}^{(4)} \quad (8)$$

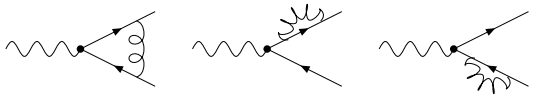
The D-dimensional metric tensor $g_{\mu\nu}^{(D)}$ acts also as a projection operator:

$$g_\mu^{(D)\rho} g_{\rho\nu}^{(4)} = g_{\mu\nu}^{(D)}. \quad (9)$$

Naive dimensional regularization is not consistent In that scheme one can derive an equation like

$$(D-4)\mathrm{Tr}\,\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5 = 0. \quad (10)$$

At $D = 4$ this equation permits the usual non-zero trace of γ_5 with four other Dirac matrices. However, for $D \neq 4$ we conclude that the trace equals zero, and there is no smooth limit $D \to 4$ which reproduces the non-zero trace at $D = 4$. A similar equation can be derived in dimensional reduction, and dimensional reduction is therefore also algebraically inconsistent. Here the inconsistency can be related to the projection property eq. 9. The 't Hooft-Veltman scheme is consistent. It should be mentioned that the 't Hooft-Veltman scheme violates the Ward identity for the axial current. This Ward identity relates the following three diagrams:



The violation of the Ward identity is not a problem per se, but one has to keep in mind, that one needs an additional finite renormalization to restore the Ward identity. To prove the (algebraic) consistency of a regularization scheme there are two approaches: The axiomatic approach of Breitenlohner and Maison [12] takes γ_μ, γ_5, $g_{\mu\nu}$ etc. as a set of abstract symbols with a given set of relations and shows that these relations are self-consistent. The constructive approach of Wilson and Collins [13] tries to find a represenation for each object in some (finite or infinite-dimensional) vector space. Here algebraic consistency is ensured by the fact that one has an explicit representation. In the following we will follow the constructive approach [14].

5. Constructing a regularization scheme

We start with a simple example. Suppose, we are given the natural numbers $1, 2, 3, ...$ together with the rule how to add two numbers and we are asked to construct or to define negative numbers. We can do that as follows: We consider pairs of natural numbers and call two pairs, (p, q) and (k, l), equivalent, if

$$p + l \quad = \quad k + q. \tag{11}$$

An addition of these pairs is defined by $(p, q) + (k, l) = (p+k, q+l)$. The set of equivalence classes yields then the integer numbers. For example, the inverse of the class of (p, q) is the class of (q, p) and the neutral element is given by the class of (p, p). Given the set of integer numbers and a rule of multiplication we can repeat the exercise and construct the rational numbers. A further example would be to consider pairs (V_i, V_j) and (V_k, V_l) from a set of vector spaces $\{V_1, V_2, V_3, ...\}$ and the equivalence relation

$$V_i \oplus V_l \quad \sim \quad V_k \oplus V_j. \tag{12}$$

This construction is quite general and forms the basis of what mathematicians call K - theory. More formally, let A be an abelian semi-group. The K-functor associates to A an abelian group $K(A)$, which is constructed as follows: Consider the equivalence relation on the set-theoretical product $A \times A$. We put $(a, b) \sim (a', b')$ when there exists a $p \in A$ such that

$$a + b' + p \quad = \quad a' + b + p. \tag{13}$$

Then by definition $K(A) = A \times A/\sim$. We are now in the position to start the construction of our regularization scheme. Let $\mathcal{V} = \{V_1, V_2, ..., V_i, ...\}$ be a set of finite-dimensional vector spaces. $\mathcal{V}$ is an abelian semi-group with respect to the direct sum $\oplus$ and the tensor product $\otimes$. We use Grothendieck's K-functor twice to construct the corresponding abelian groups and define the rank of an quadruple $[(V_i, V_j), (V_k, V_l)]$ by

$$\text{rank} \quad = \quad \frac{\dim V_i - \dim V_j}{\dim V_k - \dim V_l}. \tag{14}$$

Note the difference between the "rank" and the sum of dimensions $\dim V_i + \dim V_j + \dim V_k + \dim V_l$. We can construct a set of quadruples such that their ranks form a dense subset of $\mathbb{R}$. This construction can be extended to the complex case. Furthermore we can work with vector space of even dimension only. For a given quadruple of vector spaces of rank r and whose sum of dimension is $2m$ we define the integration by

$$\int d^r k f(k^2) = \frac{\pi^{r/2-m}}{\Gamma(r/2-m)} \int d^{2m}k \cdot \int_0^\infty dk_\perp^2 \left(k_\perp^2\right)^{r/2-m-1} f(k^2 + k_\perp^2) \tag{15}$$

The integration is well-defined, e.g. does not depend on the chosen representative. A different choice of a quadruple with the same rank r would yield the same result. In addition the definition satisfies the usual properties of an integration, e.g. linearity

$$\int d^r k (a f_1 + b f_2) = a \int d^r k f_1 + b \int d^r k f_2, \tag{16}$$

translation invariance

$$\int d^r k f(k + q) = \int d^r k f(k), \tag{17}$$

the scaling law

$$\int d^r k f(\lambda k) = \lambda^{-r} \int d^r k f(k) \tag{18}$$

and the normalization

$$\int d^r k \exp(-k^2) = \pi^{r/2}. \tag{19}$$

To define the Dirac algebra we consider

$$V_4 \oplus V_{-2\varepsilon}, \tag{20}$$

where V_4 is a four-dimensional vector space and $V_{-2\varepsilon}$ a quadruple of rank -2ε and total dimension $2m$. We recall that we can work with vector spaces of even dimension only. We now have two options to define the Dirac algebra. The first possibility is to define Dirac matrices of dimensions $2^{2+m} \times 2^{2+m}$ over $(V_4 \oplus V_{-2\varepsilon})$. With

$$\gamma_5 \quad = \quad i\gamma^0\gamma^1\gamma^2\gamma^3 \tag{21}$$

this yields the 't Hooft - Veltman scheme. The second option consists in defining Dirac matrices of dimensions $2^2 \times 2^2$ over V_4 and of dimensions $2^m \times 2^m$ over $V_{-2\varepsilon}$ separately. This corresponds to a four-dimensional scheme. By construction this scheme is free of algebraic inconsistencies. Basically the distinction between the rank and the sum of dimensions avoids the pitfall of dimensional reduction. The scheme is specified by a mapping of the Feynman rules from V_4 to $V_4 \oplus V_{-2\varepsilon}$. The Dirac algebra in the numerator can effectively be performed in four dimensions. For the loop momentum appearing in the numerator of loop integrals one has

$$\not{k}\not{k} = k^2_{(4-2\varepsilon)} - k^2_{(-2\varepsilon)} \quad (22)$$

The first term on the r.h.s can cancel a propagator, whereas the second term shifts effectively the rank of the loop integral from $4-2\varepsilon$ to $6-2\varepsilon$. As in the 't Hooft-Veltman scheme, the four-dimensional scheme violates Ward identities, which have to be restored by finite renormalizations.

6. Tensor integral reduction

Loop integrals may involve the loop momentum in the numerator. To reduce these tensor integrals to standard scalar integrals, several algorithm exists like the Pasarino-Veltman algorithm based on Lorentz invariance [15], the Feynman parameter space technique [16] or the partial integration technique [17]. These algorithm are general but have also some drawbacks: The first two introduce Gram determinants in the denominator, whereas the last one might give rise to spurious poles in $1/\varepsilon$. For most practical applications more efficient algorithm exists. For the calculation of helicity amplitudes loop momenta are usually sandwiched between spinors. Consider the example

$$\gamma^\mu \gamma^\nu \int \frac{d^D k}{(2\pi)^D} \frac{k_\mu k_\nu}{(k^2 - m_1^2)...((k-q_n)^2 - m_n^2)} \quad (23)$$

A tensor reduction according to Passarino-Veltman would result in long intermediate expressions, but rewriting

$$\gamma^\mu \gamma^\nu k_\mu k_\nu = (k^2 - m_1^2) + m_1^2 \quad (24)$$

reduces the integral immediately. More systematically, one uses the Fierz identity

$$\langle A + |\gamma_\mu| B+\rangle \langle C - |\gamma^\mu| D-\rangle = 2\,[AD]\,\langle CB\rangle, \quad (25)$$

the Schouten identity for spinors

$$\langle AB\rangle\langle CD\rangle = \langle AD\rangle\langle CB\rangle + \langle AC\rangle\langle BD\rangle, \quad (26)$$

as well as the Schouten identity for four-vectors

$$\varepsilon(q_1,q_2,q_3,q_4) k_\mu = \sum_{a=1}^{4} (k\cdot q_a)\, v_\mu^a,$$
$$v_\mu^a = \frac{1}{6}\varepsilon^{abcd}\varepsilon(\mu, q_b, q_c, q_d). \quad (27)$$

The algorithm proceeds in two steps: First one reduces tensor integrals to integrals with at most one power of the loop momentum in the numerator [18]: If there at least two massless external legs, say p_1 and p_2, one writes

$$\not{k} = \frac{1}{2p_1\cdot p_2}\left[(2k\cdot p_2)\not{p}_1 + (2k\cdot p_1)\not{p}_2 - \not{p}_1\not{k}\not{p}_2 - \not{p}_2\not{k}\not{p}_1\right]. \quad (28)$$

The first two terms will cancel propagators, whereas a product of the last two terms can be rewritten as

$$\begin{aligned}
\langle 1-|\not{k}|2-\rangle\langle 2-|\not{k}|1-\rangle &= (2k\cdot p_1)(2k\cdot p_2) \\
&\quad - (2p_1\cdot p_2)\, k^2_{(4-2\varepsilon)} + (2p_1\cdot p_2)\, k^2_{(-2\varepsilon)}, \\
\langle 1-|\not{k}|2-\rangle\langle 2+|\not{k}|1+\rangle &= \frac{1}{\langle 2-|3|1-\rangle} \\
&\quad \cdot\left[(2p_1\cdot p_3)(2p_2\cdot k)\langle 1-|\not{k}|2-\rangle\right. \\
&\quad - (2p_1\cdot k)\langle 1-|\not{p}_2\not{k}\not{p}_3|2-\rangle \\
&\quad + (2p_3\cdot k)\langle 1-|\not{p}_2\not{k}\not{p}_1|2-\rangle \\
&\quad \left. -\langle 1-|\not{p}_2\not{p}_3\not{p}_1|2-\rangle\left(k^2_{(4-2\varepsilon)} - k^2_{(-2\varepsilon)}\right)\right], \quad (29)
\end{aligned}$$

In a second step the remaining vector integrals are reduced [19]. For pentagon or higher point integrals one forms first the traces $\mathrm{Tr}_\pm \not{k}\not{p}_1\not{p}_2\not{p}_3\not{p}_4\not{p}_5$ (here $\pm$ denotes the insertion of a helicity projection operator) and uses then

$$\begin{aligned}
\mathrm{Tr}_\pm(\not{k}\not{p}_1\not{p}_2\not{p}_3\not{p}_4\not{p}_5) &= -\mathrm{Tr}_\mp(\not{p}_1\not{p}_2\not{p}_3\not{p}_4)\, k_0^2 \\
&\quad -\frac{1}{2}\left(B_0 \pm \varepsilon(k-p_1,p_2,p_3,p_4)\right) k_0^2 \\
&\quad +\frac{1}{2}\left(B_1 \pm \varepsilon(k,p_1+p_2,p_3,p_4)\right) k_1^2
\end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{2}\left(B_2 \pm \varepsilon(k, p_1, p_2+p_3, p_4)\right) k_2^2 \\
&+\frac{1}{2}\left(B_3 \pm \varepsilon(k, p_1, p_2, p_3+p_4)\right) k_3^2 \\
&-\frac{1}{2}\left(B_4 \pm \varepsilon(k, p_1, p_2, p_3)\right) k_4^2 \\
&-k^2_{(-2\varepsilon)}\left(\mathrm{Tr}_{\pm} \not{p}_1\not{p}_2\not{p}_3\not{p}_4 - \mathrm{Tr}_{\mp} \not{p}_1\not{p}_2\not{p}_3\not{p}_4\right)
\end{aligned}
\tag{30}
$$

The p_i are the external momenta, the k_i the momenta flowing through the loop propagators. For readability we dropped a subscript $(4-2\varepsilon)$ for the k_i^2. B_i depends only on the pinched integral under consideration and is given for box integrals by

$$
B_i \quad = \quad st - m_1^2 m_3^2 - m_2^2 m_4^2, \tag{31}
$$

with s and t being Mandelstam variables and the m_i denote the external masses. The derivation of eq. 30 is based on a combination of the spinor algebra approach with the dual vector approach [20]. For box integrals one can use

$$
\begin{aligned}
\mathrm{Tr}_{\pm}\left(\not{k}_0\not{p}_1\not{p}_2\not{p}_3\right) &= \frac{1}{2}\left(B \pm \varepsilon(k_0, p_1, p_2, p_3)\right) \\
&-\frac{1}{2}C_0 k_0^2 + \frac{1}{2}C_1 k_1^2 - \frac{1}{2}C_2 k_2^2 + \frac{1}{2}C_3 k_3^2
\end{aligned}
\tag{32}
$$

where C_i again depends only on the pinched integral:

$$
C_i \quad = \quad m_1^2 + m_2^2 - m_3^2. \tag{33}
$$

The m_i are the external masses of the triangle. This algorithm is most efficient, if most of the external and internal masses are zero. It does not introduce Gram determinants in the denominator. However, there might be cases where this algorithm cannot be applied (for example if all external masses are non-zero). In that case one has to resort to the general algorithms mentioned above. Luckily, these cases are in practical calculations rather rare.

7. Summary

Single top production is not only of interest for a direct measurement of V_{tb} or for a search for signals of new physics, but offers also the opportunity of polarisation studies. In this article we discussed some technical issues relevant to a corresponding NLO-calculation.

REFERENCES

1. J. van der Heide, E. Laenen, L. Phaf and S. Weinzierl, hep-ph/0003318
2. S. Catani, Y.L. Dokshitzer, M.H. Seymour and B.R. Webber, Nucl. Phys. B406, (1993), 187
3. G. Bordes and B. van Eijk, Nucl. Phys. B435, (1995), 23;
   T. Stelzer, Z. Sullivan and S. Willenbrock, Phys. Rev. D56, (1997), 5919
4. M. Smith and S. Willenbrock, Phys. Rev. D54, (1996), 6696;
   R. Pittau, Phys. Lett. B386, (1996), 397
5. B. Harris, E. Laenen, L. Phaf, Z. Sullivan and S. Weinzierl, in progress
6. G. Passarino and M. Veltman, Nucl. Phys. B160, (1979), 151
7. G. 't Hooft and M. Veltman, Nucl. Phys. B44, (1972), 189
8. S. A. Larin, Phys. Lett. B303, (1993), 113
9. W. Siegel, Phys. Lett. 84B, (1979), 193
10. Z. Bern and D.A. Kosower, Nucl. Phys. B379, (1992), 451
11. R. Kleiss and W.J. Stirling, Nucl. Phys. B262, (1985), 235
12. P. Breitenlohner and D. Maison, Comm. Math. Phys. 52, (1977), 11
13. K.G. Wilson, Phys. Rev. D7, (1973), 2911;
    J.C. Collins, Renormalization, Cambridge University Press, 1984
14. S. Weinzierl, hep-ph/9903380
15. G. Passarino and M. Veltman, Nucl. Phys. B160, (1979), 151
16. Z. Bern, L. Dixon and D.A. Kosower, Nucl. Phys. B412, (1994), 751
17. K.G. Chetyrkin and F.V Tkachov, Nucl. Phys. B192, (1981), 159
18. R. Pittau, Comp. Phys. Comm. 104, (1997), 23;
    R. Pittau, Comp. Phys. Comm. 111, (1998), 48
19. S. Weinzierl, Phys. Lett. B450, (1999), 234
20. W.L. van Neerven and J.A.M. Vermaseren, Phys. Lett. 137B, (1984), 241